INTERACTION OF ELECTRONS WITH MATTER
AGEN-625
ADVANCES IN FOOD ENGINEERING
INTERACTIONS OF ELECTRONS WITH MATTER
INTERACTIONS OF ELECTRONS WITH MATTER

- Incident electron beam
- Backscattered electrons
- X rays
- Secondary electrons
- Auger electrons
- Inelastically scattered electrons
- Direct beam
- Elastically scattered electrons
ELECTRON AND POSITRON INTERACTION

Elastic scattering

Inelastic scattering

Bremsstrahlung emission

Positron annihilation

\[ E_s = W - U_i \]
ELASTIC COLLISIONS
ELASTIC SCATTERING OF ELECTRONS WITH ATOMS AT REST

- No energy is transferred from electron to the sample
- Angular deflection is due to elastic scattering
- Assume the target has an infinite mass and does not recoil
- Energy interactions can be described as scattering of the projectile by the electrostatic field of the target
Elastic scattering of electrons with atoms at rest

- Electron penetrating into the electron cloud is attracted by the positive potential (Coulombic interactions) and its path changed.
- The closer the electron comes to the nucleus, the higher the scattering angle.
- Some time, BSE can occur.
Inelastic Collisions

\[ E_s = W - U_i \]
INELASTIC COLLISIONS

- Energy is transferred from incident electron to the sample – it produces
  - Secondary electrons
  - Photons
  - X-rays
  - Auger electrons
- Interactions that produce excitations and ionizations of the medium
- Dominant energy loss mechanism for electrons

\[ E_s = W - U_i \]

Diagram:
- Energy loss
- Incident electron (E)
- Energy loss (E-W)
- Ejected electron (E_s = W - U_i)
**INELASTIC INTERACTIONS OF ELECTRONS WITH MATTER**

- **Ionization**
  - High-energy electrons of incident beam can transfer a critical amount of energy to an inner-shell electron of an atom
  - Electron is ejected
  - Ionizing energy provided by the incident electron is reduced
  - The hole in the inner-shell can be filled up by electron from outer shell giving away part of its energy – causing emission of X-ray or Auger electrons
Bremsstrahlung emission
**Bremsstrahlung Emission**

- Electrons emit bremsstrahlung due to the acceleration caused by the electrostatic field of atoms.
- In each bremsstrahlung event, an electron with energy $E$ generates a photon of energy $W$. 

![Diagram showing an electron emitting a photon of energy $W$ with an angle $\theta$ and energy $E-W$.]
POSITRON ANNIHILATION
POSITRON ANNIHILATION

- Positrons penetrating a medium can annihilate with electrons in the medium by emission of two photons.
- Occurs when the kinetic energy $E$ of the positron is larger than the ‘absorption’ energy.
ENERGY-LOSS MECHANISM

- Beta particles can lose a large fraction of their energy and undergo large deflections in single collisions with atomic electrons.
- They do not travel in straight lines.
- A beta particle can also be sharply deflected by an atomic nucleus causing it to emit photons in the process.
Beta particle

- When it penetrates a medium, it interacts with the molecules, loses energy, and slow down
- Slow-down process – *stopping power* of the medium
STOPPING POWER

- The rate of energy loss suffered by a charged particle in traversing a unit path length
- Related to the charge and velocity of the incident particle, and physical property of the medium
- Also called the linear energy transfer (LET)
BETA PARTICLES

- Can excite and ionize atoms
- Can radiate energy by bremsstrahlung
- Can be scattered elastically by atomic electrons
Collisional Stopping Power

- Beta particle can lose a large fraction of its energy in a single collision with an atomic electron, which has equal mass.
- The identity of the beta particle and positron to atomic electrons imposes certain symmetry requirements on the equations that describe their collisions with atoms.
COLLISIONAL STOPPING POWER

\[
\left( -\frac{dE}{dx} \right)_{\text{col}}^\pm = \frac{4\pi e^4 n}{m_0 c^2 \beta^2} \left[ \ln \frac{mc^2 \tau \sqrt{\tau + 2}}{\sqrt{2I}} + F^\pm (\beta) \right]
\]

\[
F^- (\beta) = \frac{1 - \beta^2}{2} + \frac{1}{2(\tau + 1)^2} \left[ \frac{\tau^2}{8} - (2\tau + 1) \ln 2 \right]
\]

\[
F^+ (\beta) = \ln 2 - \frac{\beta^2}{24} \left[ 23 + \frac{14}{\tau + 2} + \frac{10}{(\tau + 2)^2} + \frac{4}{(\tau + 2)^3} \right]
\]

\[ e = \text{magnitude of electron charge; } n = \text{number of electrons per unit volume in the medium} \]
\[ m_0 = \text{electron rest mass; } c = \text{speed of light; } \beta = V/c = \text{speed of the particle relative to } c \]
\[ I = \text{mean excitation energy of the medium; } \tau = KE/m_0 c^2 \]
Collisional stopping power

\[
\left(- \frac{dE}{dx}\right)^\pm_{col} = \frac{5.09 \times 10^{-25} n}{\beta^2} \left[ G^\pm(\beta) - \ln I_{eV} \right] \text{ MeV/cm}
\]

\[G^\pm(\beta) = \ln(3.61 \times 10^5 \tau \sqrt{\tau + 2}) + F^\pm(\beta)\]
**Radiative Stopping Power**

- Beta particle, having little mass, can accelerate strongly by the same electromagnetic force within an atom
- Thus emitting radiation (bremsstrahlung)
- Bremsstrahlung occurs when beta particle is deflected in the electron field of a nucleus (or in the field of an atomic electron)
Radiative Stopping Power

- At high beta-particle energies, radiation is mostly emitted in the forward direction.
- Photons are produced in the direction of the electrons that produced it.

Synchrotron radiation
Bremsstrahlung Photon

- The maximum energy it can have is equal to the KE of the beta particle.
- The photon energy spectrum is approximately flat out to this maximum.
RADIACTIVE STOPPING POWER

- No single analytical formula exists for calculating the RSP
- Energy loss by radiation behaves differently from that of ionization and excitation
- The efficiency of bremsstrahlung in elements of different $Z$ varies nearly as $Z^2$
- For a given beta particle energies, bremsstrahlung losses are greater for high-$Z$ materials (lead) than in low-$Z$ materials (water)
CSP vs RSP

- Collisional energy-loss rate in an element is proportional to \( n (=\sum N_i Z_i) \), so \( Z \)
- Radiative energy-loss rate increases linearly with beta-particle energy
- Collisional rate increases only logarithmically
- At high energy, bremsstrahlung becomes the predominant mechanism of energy loss for beta-particles
# ELECTRON INTERACTION WITH WATER

<table>
<thead>
<tr>
<th>KE</th>
<th>(\beta^2)</th>
<th>CMSP [MeV cm(^2)/g]</th>
<th>RMSP [MeV cm(^2)/g]</th>
<th>TMSP [MeV cm(^2)/g]</th>
<th>Radiation yield</th>
<th>Range [g/cm(^2)]</th>
</tr>
</thead>
<tbody>
<tr>
<td>10 eV</td>
<td>4e-5</td>
<td>4</td>
<td>-</td>
<td>4</td>
<td>-</td>
<td>4e-8</td>
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<td>1.93e-2</td>
<td>42.6</td>
<td>-</td>
<td>42.6</td>
<td>-</td>
<td>8e-6</td>
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<td>0.0183</td>
<td>2.18</td>
<td>0.0416</td>
<td>4.88</td>
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<td>0.999+</td>
<td>2.40</td>
<td>26.3</td>
<td>28.7</td>
<td>0.774</td>
<td>101</td>
</tr>
</tbody>
</table>

\[
\left( - \frac{1}{\rho} \frac{dE}{dx} \right)^{\pm}_{tot} = \left( - \frac{1}{\rho} \frac{dE}{dx} \right)^{\pm}_{col} + \left( - \frac{1}{\rho} \frac{dE}{dx} \right)^{\pm}_{rad}
\]
MASS STOPPING POWER OF WATER FOR LOW-ENERGY ELECTRONS

- The radiative stopping power is negligible here.
- The end product of any form of ionizing radiation is a spatial distribution of low-energy secondary electrons, which slow down thru the energy range shown.
- It takes only 22 eV to produce secondary electron in water – so radiation produces low-energy electrons in abundance.
- 10 keV produces 450 secondary electrons, a large fraction that occur with initial energies < 100 eV.
RELATIONSHIP BETWEEN CSP & RSP

For an electron of total energy $E$, expressed in MeV in an element of atomic number $Z$:

$$\frac{(-dE/dx)_\text{rad}}{(-dE/dx)_\text{col}} \approx \frac{ZE}{800}$$

In lead, for example ($Z=82$), the two rate loss are approximately equal at a total energy given by: $82E/800 = 1$, so $E=9.8$ MeV and the electrons $KE = E-m_o c^2 = 9.3$ MeV
**Electron-photon cascade showers**

- Are the result of the dominance of radiative over collisional energy losses (at very high energies)
- High-energy beta particles emit high-energy photons
- High-energy photons produce Compton electrons and electrons-positrons pairs
- Then these produce bremsstrahlung photons
- And so on
- The electron-photon shower can initiated by either a high-energy beta particle or a photon
Radiation Yield

- The average fraction of its energy that a beta particle radiates as bremsstrahlung in slowing down completely

\[ Y \approx \frac{6 \times 10^{-4} ZT}{1 + 6 \times 10^{-4} ZT} \text{ in MeV} \]

- Bremsstrahlung can be minimum by using a shield of low-Z material to stop beta particles
- High-Z materials can be used to absorb bremsstrahlung photons
EXAMPLE

- Estimate the fraction of the energy of a 2-MeV beta ray that is converted into bremsstrahlung when the particle is absorbed in Al and in Pb

  for Al
  \[ ZT = 13 \times 2 = 26 \]
  \[ Y \approx \frac{0.016}{1.016} = 1.6\% \]

  for Pb
  \[ ZT = 82 \times 2 = 164 \]
  \[ Y \approx 9\% \]
For radiation-protection purposes, conservative assumptions can be made to apply equation for $Y$ to the absorption of beta particles from a radioactive source.

- The *maximum* beta-particle energy is used for $T$.
- This overestimates the energy converted into radiation (bremsstrahlung efficiency is less at the lower electron energies).
- Also, the assumption that all bremsstrahlung photons have the energy $T \rightarrow$ conservative estimate of the actual photon hazard.
**Example**

- A small $3.7 \times 10^8$ Bq Y$^{90}$ source is enclosed in a Pb shield just thick enough to absorb beta particles, which have a maximum energy of 2.28 MeV and an average energy of 0.94 MeV. Estimate the rate at which energy is radiated. For protection purposes, estimate the photon fluence rate at a distance of 1 m from the source.
**Solution**

\[ T = 2.28; \ Z = 82 \]

\[ Y \approx \frac{0.011}{1.01} = 0.10 \]

the total beta energy released/second

\[ (3.7 \times 10^8 \text{s}^{-1})(0.94 \text{MeV}) = 3.48 \times 10^8 \text{MeVs}^{-1} \]

rate of energy emission by bremsss.

\[ Y \times 3.48 \times 10^8 \text{MeVs}^{-1} = 3.48 \times 10^7 \text{MeVs}^{-1} \]
SOLUTION

the energy fluence rate

\[
(3.48 \times 10^7 \text{ MeVs}^{-1}) / (4\pi \times 100^2 \text{ cm}^2) = 277 \text{ MeV} / \text{ cm}^2 s
\]

For radiation hazard - assume T = 2.28 MeV

\[
227/2.28 = 121 \text{ photons/cm}^2 s
\]

For comparison, for Al (Z = 13) shield to stop the beta particles would give

\[Y \approx 0.017\]

reducing the bremsstrahlung by a factor of 5.9
**Range**

- Is the distance a beta particle travels before coming to rest
- The reciprocal of stopping power gives the distance traveled per unit energy loss

\[
R(KE) = \int_0^{KE} \left( - \frac{dE}{dx} \right)_{tot}^{\pm1} dE
\]
**Range**

- Electron ranges expressed in $g/cm^2$ are approximately the same in different materials of similar $Z$. 
The collisional mass stopping power for beta-particles is smaller in high-Z materials (Pb>H₂O).
This fact accounts for the greater range of electrons in Pb than H₂O at energies below 20 MeV.
**Range (g/cm²) – Empirical Eqn.**

- For low-Z materials
  
  - For $0.01 \leq T \leq 2.5$ MeV:
    
    $$R = 0.412(T)^{1.27 - 0.0954(T)}$$
    
    or
    
    $$\ln(T) = 6.63 - 3.24(3.290\ln R)^{1/2}$$

  - For $T > 2.5$ MeV:
    
    $$R = 0.530(T) - 0.106$$
    
    or
    
    $$T = 1.89R + 0.200$$
Beta rays range

- Range is greater than the thickness of the epidermis.
- A 70 keV electron can penetrate the minimum thickness of 7 mg/cm² of the epidermal layer.
- ⁹⁰Y emits a beta particle with a max. energy of 2.27 MeV (range over 1 g/cm²).
- Beta emitters can damage the skin and eyes (also internal radiation hazard).
Example

- How much energy does a 2.2MeV electron lose in passing through 5 nm of Lucite (density = 1.19 g/cm$^3$)?
SOLUTION

- Lucite is a low-Z material

\[ R = 0.412(2.2)^{1.27-0.0954(2.2)} \]

\[ R = 1.06 \text{ g/cm}^2 \]

\[ d = \frac{R}{\rho} = \frac{1.06 \text{ g/cm}^2}{1.19 \text{ g/cm}^3} = 0.891 \text{ cm} \]
Since the Lucite is only 0.5 cm thick, the electron emerges with enough energy $T'$ to carry it another 0.391 cm or 0.465 g/cm². The energy $T'$ is:

$$\ln T' = 6.63 - 3.24(3.29 - \ln 0.465)^{1/2} = 0.105$$

and so $T' = 1.11$ MeV

The energy lost by the electrons is $T - T' = 2.2 - 1.11 = 1.09$ MeV
SLOWING-DOWN RATE & TIME

- The total stopping-power formula can be used to calculate the rate at which beta particles slow down.
- The time rate of energy loss, \(-\frac{dE}{dt}\) can be expressed in terms of stopping power as:

\[-\frac{dE}{dt} = (-\frac{dE}{dx})(\frac{dx}{dt}) = V(-\frac{dE}{dx})\]

where \(V = \frac{dx}{dt}\) is the velocity of the particle.
- The stopping time is calculated as the ratio of the initial energy and the total slowing-down rate:

\[\tau = \frac{T}{(-\frac{dE}{dt})}\]
**Examples of Electron Tracks in Water**

- Typical EGS-generated tracks
- Showing bremsstrahlung X-rays being produced by electrons
CHARACTERISTICS OF ELECTRONS TRACKS

- The tracks tend to wander – large deflections electrons experience in single collision
- Wandering is augmented at low energies (end of the track) by increased elastic scattering
- Energy-loss events are more sparsely distributed at the beginning of the track (primary electrons move faster)
EXAMPLE OF ELECTRON SHOWER

- EGS simulation of 1 GeV electron shower in 15 cm of Cu (10 incident)
SINGLE-COLLISION SPECTRA IN WATER

- Understanding the interaction of low-energy electrons with matter is fundamental to understand the physical and biological effects of ionizing radiation.
- The abundant low-energy electrons are responsible for producing the initial alterations that lead to chemical changes in biological materials, such as water.
SINGLE-COLLISION SPECTRA IN WATER

- The interaction of an electron of kinetic energy $KE$ is the probability $N(T,E)dE$ that it loses an amount of energy between $E$ and $E+dE$ in a single collision.
- The distribution $N(T,E)$ is called the single-collision spectrum for an electron of energy $KE$.
- As a probability function, it is normalized:

$$Q_{\text{max}} \int_{0}^{Q_{\text{max}}} N(T,E)dE = 1$$

it has the dimensions of inverse energy.
SINGLE-COLLISION SPECTRA OF ELECTRONS IN WATER

- Curves reflect the basic physics of electron interactions with water.
- For 10 keV electrons, the average value of the spectrum for energy losses between 45 and 50 eV is 0.01 eV$^{-1}$.
- The relative # of collisions between these energy losses is $5\text{eV} \times 0.01\text{eV}^{-1} = 0.05$ (5\% change of a 10 keV electron in water loses energy between 45-50 eV in its next collision).
SINGLE-COLLISION SPECTRA OF ELECTRONS IN WATER

- All curves start at an estimated energy threshold of 7.4 eV (minimum energy required for electron excitation)
- Excitation takes place to bound (see the resonance peaks at low $E$)
- Spectrum of energy losses due to ionization is continuous – not restricted to discrete values
**Probability of a Given Energy-Loss (Ionization or Excitation)**

- Energy loss events will cause ionization or excitation?
- Ionization increases very rapidly with electron energy.
- $T > 150$ eV, almost 95% of the energy losses result in ionization.
**AVERAGE ENERGY LOST**

- Collisional stopping power is related to $N(T,E)$
- The average energy lost by an electron is:
  \[
  \overline{E}(T) = \int_{0}^{Q_{\text{max}}} EN(T, E)\,dE
  \]
- The stopping power is:
  \[
  \left(-\frac{dE}{dx}\right)_{\text{tot}}^{-} = \mu(T)\overline{E}(T) = \mu(T) \int_{0}^{Q_{\text{max}}} EN(T, E)\,dE
  \]

Probability per unit distance that an inelastic collision occurs
**Delta Rays**

- Are secondary electrons produced by an electron traversing the matter.
- Secondary electrons have enough energy to leave the path of the primary electron producing a track of its own.
ENERGY LOST AND ENERGY ABSORBED

- Stopping power gives energy *lost* by a charged particle in a medium.
- Radiation dose is the energy *absorbed* by unit mass in an irradiated material.
- Energy lost and energy absorbed are not the same (specially if target is small compared with ranges of delta rays produced).
- Many living cells have diameter in the order of microns, subcellular structures (DNA – 20 A) can be many times smaller.
- Delta rays can effectively transport energy out of the original site in which it is lost by a primary particle.
RESTRICTED STOPPING POWER

- It is the linear rate of energy loss due only to collisions in which the energy transfer does not exceed a specific value of $\Delta$:

$$\left(-\frac{dE}{dx}\right)_{\Delta}^{-} = \mu(KE)\int_{0}^{\Delta} EN(KE, E)dE$$

- $\Delta$ can be selected as 100eV, 1 keV, etc.

- To associate energy loss in a target more closely with the energy that is actually absorbed there
Restricting single-collision energy losses by electrons in water to 100eV or less limits the range of secondary electrons to 5x10^{-7} cm.

With $\Delta = 5$ keV the maximum range of 2nd electrons contributing to the restricted SP is 8x10^{-5} cm.
EXAMPLE

A sample of bacteria, assumed to be in the shape of spheres of diameter 300 Å (3x10^{-6} cm), is to be irradiated by a charged particle beam. Estimate the cutoff value that would be appropriate for determining a restricted SP that would be indicative of the actual energy absorbed in the individual bacterium particles.
SOLUTION

- As an approximation – specify that the range of most energetic delta rays should not exceed 300 Å
- Assume that the bacteria sample has unit density
- Table 6.1: this distance is approximately the range of 700-eV power
- We choose $\Delta = 700$ eV and use $-(dE/dx)_{700eV}$ as a measure of the average energy absorbed in an individual bacterium particle from a charged particle traversing it
## RMSP of Water for Protons

<table>
<thead>
<tr>
<th>Energy [MeV]</th>
<th>(-\left(\frac{dE}{\rho dx}\right)_{100\text{ eV}})</th>
<th>(-\left(\frac{dE}{\rho dx}\right)_{1\text{ keV}})</th>
<th>(-\left(\frac{dE}{\rho dx}\right)_{10\text{ keV}})</th>
<th>(-\left(\frac{dE}{\rho dx}\right)_{\infty})</th>
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## RCMSP of Water for Electrons

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<th>Energy [MeV]</th>
<th>$-\left( \frac{dE}{\rho dx} \right)_{100,eV}$</th>
<th>$-\left( \frac{dE}{\rho dx} \right)_{1,eV}$</th>
<th>$-\left( \frac{dE}{\rho dx} \right)_{10,eV}$</th>
<th>$-\left( \frac{dE}{\rho dx} \right)_{\infty}$</th>
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<td>1.1</td>
<td>1.3</td>
<td>1.5</td>
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</table>
RESTRICTED LINEAR ENERGY TRANSFER

- The restricted stopping power for energy losses not exceeding $\Delta$:

$$LET_\Delta = -\left(\frac{dE}{dx}\right)_\Delta$$
EXAMPLE

- Use table 7.1 to determine LET (1keV) and LET (5keV) for 1-MeV protons in water
SOLUTION

- Note that previous equation for LET involves SP rather than MSP
- Since density = 1 g/cm³ for water
- LET$_{1\text{keV}} = 238$ MeV/cm
- LET$_{1\text{keV}} = \text{linear interpolation} = 252$ MeV/cm
**Specific Ionization**

- It is the average number of ion pairs that a particle produces per unit distance traveled.
- It expresses the density of ionizations along a track.
- The specific ionization of a particle at a given energy is equal to the stopping power divided by the average energy required to produce an ion pair at that particle energy.
**Specific Ionization**

- The SP of air for a 5-MeV alpha particle is 1.23 MeV/cm
- An average of about 36 eV is needed to produce an ion pair
- The SI is \( \frac{1.23 \times 10^6 \text{ eV/cm}}{36 \text{ eV}} = 34,200 \text{ cm}^{-1} \)
- For 5-MeV alpha particle in water \( \frac{\text{d}E}{\text{d}x} = 950 \text{ MeV/cm} \) (about 22 eV required to produce an ion pair, so SI = \( 4.32 \times 10^7 \text{ cm}^{-1} \))
Energy Straggling

- It is the phenomenon of unequal energy losses under identical condition.
- As a charged particle penetrates matter, statistical fluctuations occur in the number of collision along its track and in the amount of energy lost in each collision.
- The existence of different path lengths is called range straggling.
MULTIPLE COULOMB SCATTERING

- The path of a charged particle in matter deviates from a straight line because it undergoes frequent small-angle nuclear scattering events.
- A heavy particle deviates repeatedly by multiple Coulomb scattering.
- The pathlength traveled $R$ is greater than the depth of penetration $x_0$. 

\[ R > x_0 \]
MULTIPLE COULOMB SCATTERING

- Another example is the spread of a pencil beam of charged particles as it penetrates a target.
- The magnitude of the spreading increases with the atomic number of the material.